Therefore, the only possible solutions are

$$xu-5yv=2, \quad xu+5yv=\pm 2.$$

Whence, $2xu=4$ or 0, $10yv=0$ or -4; *i. e.*, either x, u, y, or v equals zero.

Solved, similarly, by G. B. M. Zerr and V. M. Spunar.

II. Solution by O. C. CARMICHAEL, Oxford, Ala.

If the square of the first equation be added to five times the square of the second equation, we have

$$x^2u^2+5x^2v^2+25y^2v^2+5y^2u^2=9.$$

Therefore, there is no integral solution in x, y, u, v except when one of the unknowns is zero; for, if they were all positive integers, $25y^2v^2$ itself would be greater than 9.

303. Proposed by PROF. R. D. CARMICHAEL, Anniston, Ala.

Evaluate the determinant

$$\begin{vmatrix} D_1 & x_1x_2 & x_1x_3 & \dots & x_1x_n \\ x_1x_2 & D_2 & x_2x_3 & \dots & x_2x_n \\ x_1x_3 & x_2x_3 & D_3 & \dots & x_3x_n \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ x_1x_n & x_2x_n & x_3x_n & \dots & D_n \end{vmatrix}$$

Solution by J. W. CLAWSON, Ursinus College, Collegeville, Pa.

$$\triangle = x_1x_2x_3\dots x_n \begin{vmatrix} D_1/x_1 & x_2 & x_3 & \dots & x_n \\ x_1 & D_2/x_2 & x_3 & \dots & x_n \\ x_1 & x_2 & D_3/x_3 & \dots & x_n \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ x_1 & x_2 & x_3 & \dots & D_n/x_n \end{vmatrix}$$

$$= \prod_1^n x_1 \begin{vmatrix} (D_1-x_1^2)/x_1 & 0 & 0 & \dots & (x_n^2-D_n)/x_n \\ 0 & (D_2-x_2^2)/x_2 & 0 & \dots & (x_n^2-D_n)/x_n \\ 0 & 0 & (D_3-x_3^2)/x_3 & \dots & (x_n^2-D_n)/x_n \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ x_1 & x_2 & x_3 & \dots & D_n/x_n \end{vmatrix}$$

Subtracting the last row from each row,

$$=\prod_1^n x_r.\prod_1^n \frac{D_r-x_r^2}{x_r}\begin{vmatrix} 1 & 0 & 0 & \dots & -1 \\ 0 & 1 & 0 & \dots & -1 \\ 0 & 0 & 1 & \dots & -1 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \frac{x_1^2}{D_1-x_1^2} & \frac{x_2^2}{D_2-x_2^2} & \frac{x_3^2}{D_3-x_3^2} & \dots & \frac{D_n}{D_n-x_n^2} \end{vmatrix}$$

$$=\prod_1^n (D_r-x_r^2\begin{vmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & -1 \\ 0 & 0 & 1 & \dots & -1 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \frac{x_1^2}{D_1-x_1^2} & \frac{x_2^2}{D_2-x_2^2} & \frac{x_3^2}{D_3-x_3^2} & \dots & \frac{D_n}{D_n-x_n^2}+\frac{x_1^2}{D_1-x_1^2} \end{vmatrix}$$

adding last column to the first. Since the first row contains $n-1$ zeros, this reduces the determinant to one of the $(n-1)$st order. Adding last column to the first we reduce to one of the $(n-2)$nd order. Repeat this process $(n-1)$ times in all.

$$\text{Then } \triangle=\prod_1^n (D_r-x_r^2)\left[\frac{D_n}{D_n-x_n^2}+\frac{x_1^2}{D_1-x_1^2}+\frac{x_2^2}{D_2-x_2^2}+\dots\frac{x_{n-1}^2}{D_{n-1}-x_{n-1}^2}\right]$$

$$=\prod_1^n (D_r-x_r^2)\left[\frac{x_1^2}{D_1-x_1^2}+\frac{x_2^2}{D_2-x_2^2}+\dots\frac{x_{n-1}^2}{D_{n-1}-x_{n-1}^2}+\frac{x_n^2}{D_n-x_n^2}+1\right]$$

$$=\prod_1^n (D_r-x_r^2)\left[\sum_1^n\frac{x_r^2}{D_r-x_r^2}+1\right].$$

Also solved by G. B. M. Zerr, V. M. Spunar, and J. Scheffer.

304. Proposed by C. N. SCHMALL, New York City.

A policeman on a motor-cycle starts in pursuit of an automobile when the latter has a headway of $\frac{1}{2}$ a mile. A pedestrian who is $\frac{1}{4}$ mile ahead of the auto and who is walking at the rate of 5 miles an hour, notices that when the auto overtakes him the policeman is only 5-12 of a mile behind the auto, and $2\frac{1}{2}$ miles from where the officer started; he overtakes the auto. How long did the chase last?

Solution by G. B. M. ZERR, A. M., Ph. D., and the PROPOSER.

Let x=policeman's rate, y=auto's rate, and z=time for auto to overtake pedestrian.

Then $(5z+\frac{1}{4})/y=(5z+\frac{1}{4}+\frac{1}{12})/x\ldots(1)$,

$2/y=(2\frac{1}{2})/x\ldots(2)$.

(1)/(2) gives $300z+15=240z+16$. $\therefore z=\frac{1}{60}$ hours=1 minute.